1. What is meant by optimization? Give some examples of its applications.

Definition of Optimization

- In engineering, physical, and mathematical sciences, computational optimization, or simply optimization, means either minimization or maximization of a certain objective function.

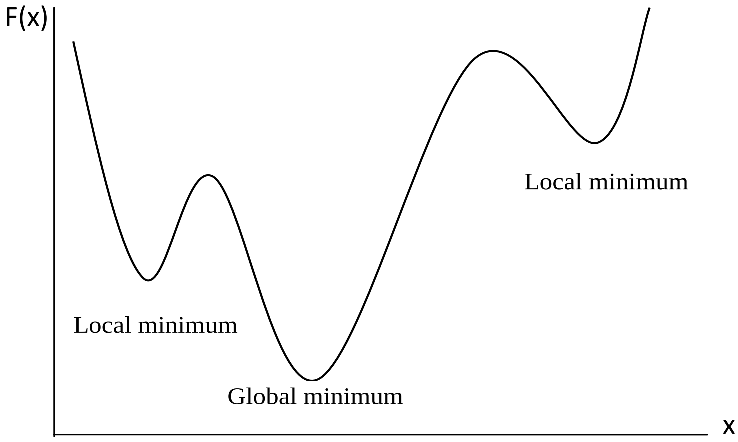
Examples of Optimization Problems/ Applications

- Optimizing the parameters of the ANN model (weights and biases).
- Optimizing the parameters of the ANFIS model (premise and antecedent parameters).
- Tuning the parameters of the PID controller (get the best value for K_P, K_I and K_D).
- Getting the best Placement of Wi-Fi Access Point for Indoor Positioning system (IPS).
- Approximating experimental, mathematical functions with the least possible error.
- Optimizing the methods of Micro-array data analysis (Bioinformatics field).

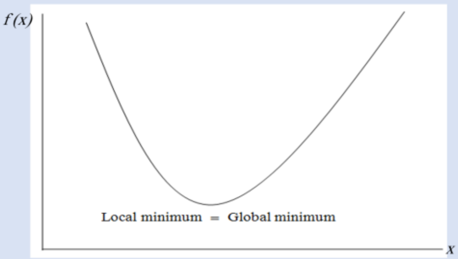
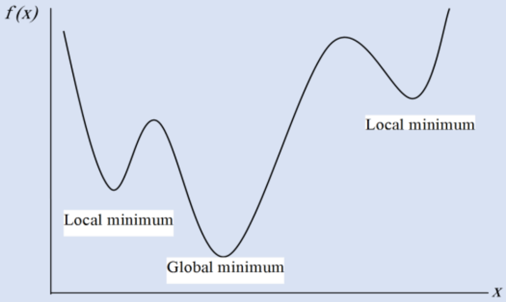
2. Explain the main differences between the following terms:

➢ Global optimum and local optimum

- Optimum can be global or local. To illustrate, the single-variable function $f(x)$ (represents the objective function to be minimized) in Fig.1 has one global minimum and two local minimum; the global minimum is the least among all local minimum.
- **Note that** every global optimum is a local optimum, but the reverse is not necessarily true.



➢ Unimodal and multimodal functions

Unimodal functions	multimodal functions
A unimodal function has a single local optimum which is itself the global optimum  Fig. 2: $f\ (X)$ is a unimodal function	A multimodal function has more than one local optimum and one global optimum which is a local optimum with the least objective function value among all other local optimum  Fig. 1: $f\ (X)$ is a multimodal function

➢ Separable and non-separable functions

Separable functions	Non-separable functions
If function $\boldsymbol{f}\ (\boldsymbol{x}_1,\ \boldsymbol{x}_2, \ldots\ldots., \boldsymbol{x}_D)$ can be divided to D functions **in the following form**: $\boldsymbol{f}\ (\boldsymbol{x}_1) + \boldsymbol{f}(\ \boldsymbol{x}_2) + \ \ldots\ldots \boldsymbol{f}(\boldsymbol{x}_D)$	If function $f\ (x_1,\ x_2, \ldots\ldots., x_D)$ can be divided to D functions in the following form: $f\ (x_1) + f(\ x_2) + \ \ldots\ldots f\ (x_D)$

➢ <u>Univariate and Multivariate</u>

Univariate	Multivariate
Number D of variables x_d (Dimensionality) (D = 1)	Number D of variables x_d (Dimensionality) (D > 1)

➢ <u>Traditional optimization methods and population-based optimization methods</u>

Evolutionary optimization algorithms are <u>population-based algorithms</u> of candidate solutions, not just one solution as in traditional methods.

➢ <u>Exploration and exploitation</u>

Exploration	Exploitation
Exploration means finding new solutions (or points) in the search domains which have not been evaluated before. In exploration, the variation of the population members from one iteration to another is large.	Exploitation means trying to improve the current found solutions by performing relatively small changes that lead to new solutions in the immediate neighborhood. In exploitation, the variation of the population members from one iteration to another is very small.

3. <u>How can we evaluate an optimization algorithm?</u>

- In order to evaluate the <u>efficiency</u> and <u>robustness</u> for an optimization algorithm, standard complex mathematical functions with different characteristics called **benchmark functions** are used to test the optimization algorithm.
- After selecting a suitable set of benchmark functions, the algorithm is running over these functions for N independent of runs. Each run consists of determined No. of iterations.
- The results of the test show the No. of successful runs for each function. The run is considered successful if the algorithm reached to the required global optimum.

4. <u>What are the basic elements of the optimization process?</u>

(1) An objective function *f* which is the function to be optimized (minimized or maximized).

(2) The number of components or variables of the objective function that specifies <u>the dimensionality of the optimization problem</u>

If the objective function f is expressed in the form $f(x_1, x_2, \ldots, x_D)$

Then $x_1, x_2, \ldots, x_D$ are the independent variables and D is the number of variables specifies the dimensionality of the problem.

The objective function can be written compactly as:

F(x) , x = [$x_1, x_2, \ldots, x_D$]is a 1xD vector

(3) A set of constraints forced on the required solution

Most problems constrain at least the search domains of the variables vector

x = [$x_1, x_2, \ldots, x_D$]

5. State whether the following sentences is true (T) or false (F) and correct the wrong ones:

➢ **Every local optimum is a global optimum.** (T/**F**) False

Correction: Every global optimum is a local optimum, but the reverse is not necessarily true.

➢ **The exploitation process is done before exploration process.** (T/**F**) False

Correction: The exploration process is done before exploitation process.

➢ **Large number of solutions in the populations results in decreasing the exploration rate.** (T/**F**) False

Correction: Large number of solutions in the populations results in increasing the exploration rate.

➢ **Any optimization algorithm starts with large exploration rate.** (**T**/F) True

➢ **The more No. of successful runs for an optimization algorithm over a certain objective function, the less robustness of the optimizer.** (T/**F**) False

Correction: The more No. of successful runs for an optimization algorithm over a certain objective function, the more robustness of the optimizer.

➢ **The goal of the optimizer is to reach to a local optimum.** (T/**F**) False

Correction: The goal of the optimizer is to reach to a global optimum.

➢ **A 'good' optimizer should get trapped in only one local optimum.** (T/**F**) False

Correction: A 'good' optimizer should get trapped in only one global optimum.

➢ **The No. of objective functions to be optimized specifies the dimensionality of the optimization problem**. (T/**F**) False

Correction: The number of components or variables of the objective function that specifies the dimensionality of the optimization problem

➢ **The minimum value of *f* (x) is the same as the maximum value of -*f* (x).** (T/**F**) False

Correction: The minimum value of *f* (x) is the same as the negative maximum value of -*f*(x).

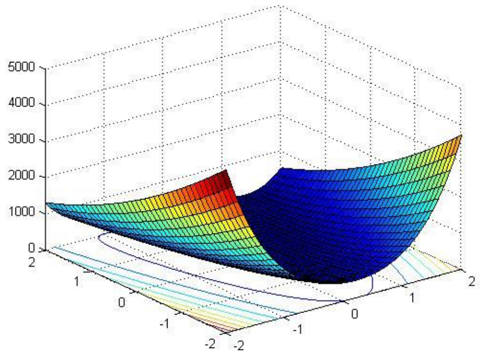
6. Write a MATLAB code to program to draw the 3-D map of the following benchmark functions:

Benchmark Functions		Search Range
Rosenbrock Function	$f_1(x) = \sum_{i=1}^{D-1} [100(x_i^2 - x_{i+1})^2 + (x_i - 1)^2]$	$[-2.048, 2.048]^D$
Griewank Function	$f_2(x) = 1 + \sum_{i=1}^{D} \frac{x_i^2}{4000} - \prod_{i=1}^{D} \cos\left(\frac{x_i}{\sqrt{i}}\right)$	$[-600, 600]^D$
Rastrigin Function	$f_3(x) = \sum_{i=1}^{D} [10 + x_i^2 - 10\cos(2\pi x_i)]$	$[-5.12, 5.12]^D$
Schwefel Function	$f_4(x) = 418.9829\, D - \sum_{i=1}^{D} x_i \sin(\sqrt{\lvert x_i \rvert})$	$[-500, 500]^D$

(Hint: Put D = 2 to draw the 3-D maps)

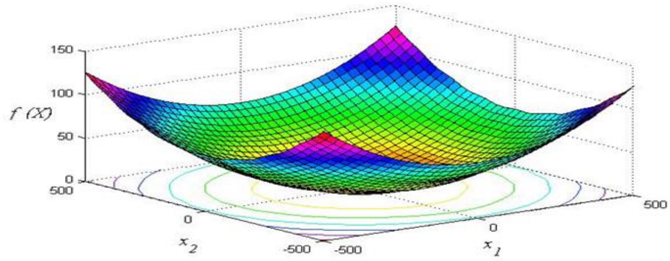
- **<u>Rosenbrock Function</u>**

```
>> x=[-2:0.1:2];
>> [x1 , x2] = meshgrid(x);
>> z=100*((x1.^2 - x2).^2 +(x1 - 1).^2);
>> surfc(x1,x2,z)
>> colormap(hsv)
```



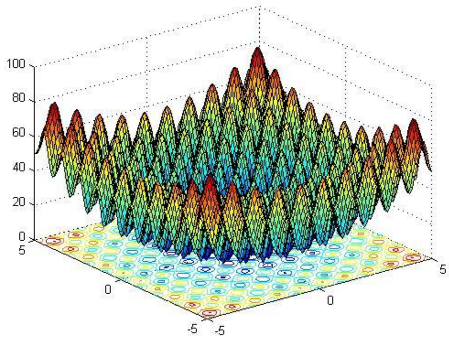
- **<u>Griewank Function</u>**

```
>> x=[-500:1:500];
>> [x1 , x2] = meshgrid(x);
>> z = 1 + (x1.^2)./4000 +(x2.^2)./4000 - (cos(x1./sqrt(1))*cos(x2./sqrt(2)));
>> surfc(x1,x2,z)
>> colormap(hsv)
```



- **Rastrigin Function**

```
>> x=[-5:0.1:5];
>> [x1 , x2] = meshgrid(x);
>> z=(10 + x1.^2 - 10*(cos(2*pi.*x1))) + (10 + x2.^2 - 10*(cos(2*pi.*x2)));
>> surfc(x1,x2,z)
```



- **Schwefel Function**

```
>> x=[-500:1:500];
>> [x1 , x2] = meshgrid(x);
>> z = (418.9829*2) - ((x1.*sin(sqrt(abs(x1)))) + (x2.*sin(sqrt(abs(x2)))));
>> surfc(x1,x2,z)
>> colormap(hsv)
```